<u>A Relation between Filter Parameters of the Tow-Thomas-Biquad</u>

In this paper a relation between several filter parameters of the often empolyed Tow-Thomas-Biquad will be derived, with the aim to make usually necessary compromises between these filter parameters comprehensible. Filters at higher frequency and/or high quality factors are of special interest here.

Suppose the biquad is configured as a bandpass according to the following signal flow:

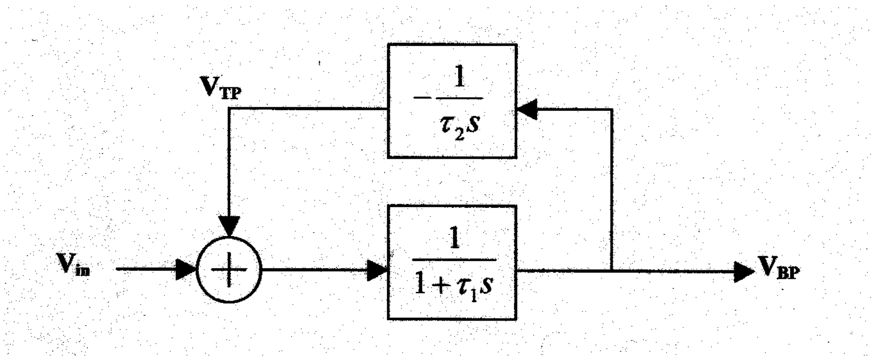

Fig. 1

The filter therefore possesses the transfer function

$$\frac{V_{BP}}{V_{in}} = H(s) = \frac{t_2 s}{1 + t_2 s + t_1 t_2 s^2}$$

$$= \frac{\frac{s}{w_m Q}}{1 + \frac{s}{w_m Q} + \frac{s^2}{w^2{}_m}}$$

With the center frequency ω_m and the quality factor Q and the bandwidth B= f_m / Q, as well as the passband gain A_0 =1. Now imagine n poles inserted into the feedback path at ω_{pn} , representing the frequency deviation of the used operational amplifiers. In a strict sense, the poles should have been distributed to the feedback *and* forward path. Poles in the forward path become poles of the total transfer function. But if the pole frequencies lie magnitudes above the center frequency ω_m, poles not in the feedback loop influence the frequency deviation of the filter only marginally in practice.

At frequencies far below the bandwidth of the used operational amplifiers one can, as an approximation of first order, replace all poles by one dominant pole:

$$\frac{1}{1+j\Omega/\Omega_{p1}} \cdots \frac{1}{1+j\Omega/\Omega_{pn}}$$

$$\approx \frac{1}{1+j\Omega(\frac{1}{\Omega_{p1}}+\ldots+\frac{1}{\Omega_{pn}})} = \frac{1}{1+j\Omega/\Omega_p}$$

with the normalized frequency $\Omega = \omega/\omega_m$ and the normalized pole angular frequencies Ω_{pl}, ...Ω_{pn}, as well as the normalized substitute angular frequency Ω_p (all referenced to ω_m).
The total transfer function in $j\Omega$ is then:

$$\hat{H}(j\Omega) = \frac{j\Omega/Q}{\frac{1}{1+j\Omega/\Omega_p} + j\Omega/Q - \Omega^2}$$

For $\Omega_p >> \Omega$: $\frac{1}{1+j\Omega/\Omega_p} \approx 1 - j\Omega/\Omega_p$ the following approximation is valid:

$$\hat{H}(j\Omega) = \frac{j\Omega/Q}{1+j\Omega(\frac{1}{Q}-\frac{1}{\Omega_p})-\Omega^2}$$

$$= \hat{A}_0 \frac{j\Omega/\hat{Q}}{1+j\Omega/\hat{Q}-\Omega^2}$$

with $\frac{1}{\hat{Q}} = \frac{1}{Q} - \frac{1}{\Omega_p}$, and therefore

$$\boxed{\frac{\hat{Q}}{Q} = \frac{1}{1-\frac{w_m}{w_p}}} \quad \text{'Q enhancement'}$$

and $$\boxed{\hat{A}_0 = A_0 \frac{\hat{Q}}{Q}} \quad \text{'Amplitude enhancement'}$$

The transfer function of the filter with additional poles in the feedback is therefore, under the given assumptions for approximation, again a bandpass function of second order with the same center frequency, though with a higher Q and a higher passband gain. (A more exact analysis would show a shift of the center frequency also.)

The Tow-Thomas-Biquad generally consists of three operational amplifiers. If we assume for all operational amplifiers a frequency response of

$$H_{Amp}(j\Omega)=\frac{A_{Amp}}{1+\frac{j\Omega}{\Omega_{Amp}}}$$

we get

$$w_p \approx \frac{w_{Amp}}{3}$$

Should we need a deviation of the passband gain of one being smaller than 1 dB, and respectively a tolerance of Q of maximum ca. 10% , we would have to require

$$\frac{1}{1-Q\frac{3w_m}{w_{Amp}}} \leq 1{,}12$$

leading to the following demand on the operational amplifier bandwidth

$$\boxed{f_{Amp} \geq 30 \cdot Q \cdot f_m}$$

(f_m is centerfrequency)

Requiring a maximum deviation of bandpass gain of
$\frac{\hat{A}_0}{A_0} \leq 1+d$ one gets:

$$\boxed{f_{Amp} \geq 3 \cdot (1+\frac{1}{d}) \cdot Q \cdot f_m}$$

For very small δ though, the decrease in passband gain by the poles in the forward path has to be considered.
One percieves how the OpAmp bandwidth has to increase with Q to maintain filter precision!

Now we want to consider dynamic also. One can see from fig. 1, that V_{TP}/V_{in} represents a lowpass transfer function with w_m, Q. So one has

$$\max\frac{|V_{TP}|}{|V_{in}|}=\frac{Q}{\sqrt{1-\frac{1}{4Q^2}}} \approx Q$$

the approximation being valid for higher quality factors $Q \geq 2$.

Because the output voltage of the operational amplifiers sets an upper limit to the dynamic range and the highest signal voltage (or highest signal current, depending on the implemetation) is to be found at V_{TP}, it makes sense to indroduce some quantity D (the name should remind of dynamic range), by

$$D := \min_f \frac{|V_{in}|}{|V_{max}|}$$

$|V_{max}|$ should be the smallest output voltage limit of the amplifiers used. Obviously D will give no information whatsoever about the lower limit of the dynamic range because it doesn't consider noise! But D will indicate directly to what extend input and output voltage range of the operational amplifiers can be utilized. Over the whole frequency range it has to be at least (rail to rail operation) in fig. 1:

$$V_{in} \le D \cdot V_B / \sqrt{2}$$

with V_{in} being the effective voltage of a sinus signal at the input and V_B the minimum of positive and negative supply voltage.

This leads to a second general condition (approx. for high Q)

$$\boxed{Q \cdot D \le const}$$

wich provides a relation between quality factor and dynamic range in the sense of a maximum allowable (input or output) level. "const" depends on the chosen implementation, as will be shown in the following example.

Let us now take a look at a circuit implementation of a Tow-Thomas-bandpass :

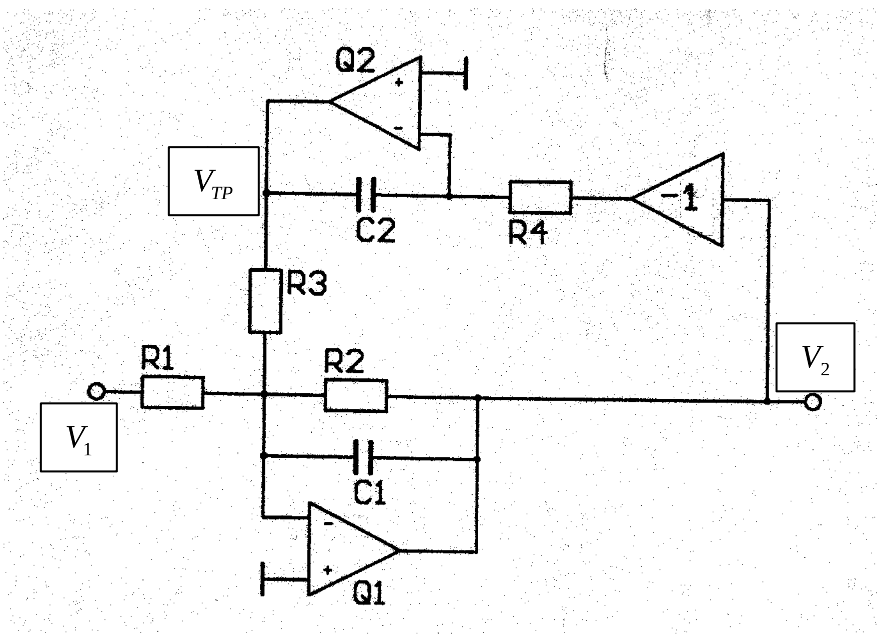

Abb.

For the transfer function one gets

$$V_2 = -\frac{R_2}{R_1} \cdot \frac{jwC_2R_3R_4/R_2}{1 + jwC_2R_3R_4/R_2 - w^2C_1C_2R_3R_4} \cdot V_{in}$$

and for the output voltage of the lowpass contained in the feedback path:

$$V_{TP} = -\frac{R_3}{R_1} \cdot \frac{1}{1 + jwC_2R_3R_4/R_2 - w^2C_1C_2R_3R_4} \cdot V_{in}$$

The formulas yield a passband gain of $A_0 = -R_2/R_1$ and therefore

$$\max \frac{|V_{TP}|}{|V_{in}|} \approx \frac{R_3}{R_1} Q .$$

(Again for higher quality factors, meaning about Q≥2)

Consequently a useful definition of D would be $D = \frac{1}{Q} \cdot \frac{R_1}{R_3}$.

The damped integrator provides DC-gain also. The feedback factor is given by

$$k = \frac{1}{1 + \frac{R_2}{R_1 \parallel R_3}}$$

So for higher gain>>1 the bandwidth of the amplifier in the forward path of the filter would have to be enhanced by the factor 1/k to keep the phase deviation from worsening. This is at least true for voltage feedback amplifiers. But even current feedback amplifiers won't help much. Because of the capacitor present at the inverting input, a resistor would have to be inserted between the node R_1, R_3, R_2, C_1 and the inverting input of the amplifier – which again would make the bandwidth depend on k respectively R_1 and R_3 !

Further follows with $-\frac{R_2}{R_1} = A_0$ as passband gain in this realization:

$$\begin{aligned} \frac{1}{k} &= 1 + \frac{R_2}{R_1 \parallel R_3} \\ &= 1 + \frac{R_2}{R_1}\left(1 + \frac{R_1}{R_3}\right) \\ &= 1 + |A_0| \cdot (1 + D \cdot Q) \end{aligned}$$

Inserted into the relation found earlier for the OpAmp bandwidth

$$f_{Amp} \geq 3 \cdot \left(1 + \frac{1}{d}\right) \cdot \left[1 + |A_0|(1 + Q \cdot D)\right] \cdot Q \cdot f_m$$

respectively for $Q \cdot D = \frac{R_1}{R_3} >> 1, \quad d << 1,$ the estimation

$$\boxed{f_{Amp,\text{int}} \geq 3 \cdot \frac{1}{d} \cdot |A_0| \cdot D \cdot Q^2 \cdot f_m}$$

may give at least some qualitative relation between the different parameters. Being – after so many steps of simplification – some way from truth, it is recommended to simulate the circuit for given amplifiers in any case after having done the first iterations and estimates by hand calculation !

Conclusion:

It should have become clear by now, what compromises between center frequency, quality factor, precision, passband gain and bandwidth have to be made. Practical experience indeed will show what above formulas lead to suspect: For high Q *and* high center frequencies the Tow-Thomas-Biquad may be no longer the right choice of circuit to realize a high quality bandpass. That is true especially for the use as filterblock in more complex filters, e.g. a FLF-structure. Either one uses another resonator topology or partitions the filter further by using filter blocks with lower pole Q.
Similar relations can – by the way – be derived for other filter types like Sallen-Key.